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Working Paper Number	1982-001A
Creation Date	January 1982
Citable Link	https://doi.org/10.20955/wp.1982.001
Suggested Citation	Davidson, L.S., Hafer, R.W., Hein, S.E., 1982; Econometric Limitations of Fama's Interest Rate and Inflationary Expectations Framework, Federal Reserve Bank of St. Louis Working Paper 1982-001. URL https://doi.org/10.20955/wp.1982.001

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We would like to thank Richard Froyen, Bill Kelly and Mack Ott for helpful comments on an earlier draft. Any errors or omissions remain the author's responsibility.

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ECONOMETRIC LIMITATIONS OF FAMA'S INTEREST RATE AND INFLATIONARY EXPECTATION FRAMEWORK

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In an innovative and controversial paper examining the link between interest rates and expected inflation, Eugene Fama concluded that "one cannot reject the hypothesis that all variation through time in one- to six-month nominal rates of interest mirrors variation in correctly assessed one- to six-month expected rates of change in purchasing power."[1/] Even though this conclusion has been widely disputed, the general approach has been to accept Fama's framework of analysis and consider alternative and stronger tests which lead to rejection of the null hypothesis.[2/]

This comment takes a different tack: It is argued that Fama's framework and conclusions are seriously limited and that once key restrictions are relaxed, serious questions with regard to the usefulness of this framework are raised. In fact, the framework lacks sufficient robustness to allow for the random walk in the expected real rate that Fama himself has recently suggested. Moreover, correcting for these econometric concerns requires a specification of inflationary expectations, a task which Fama apparently had successfully avoided.

Fama's Original Model

Fama's model of the linkage between interest rates and inflation expectations consists of three equations:

$$(1) \quad \dot{P}_{t+1} = {}_t\dot{P}^e_{t+1} + \mu_{t+1}$$

$$(2) \quad i_t = {}_t r^e_{t+1} + {}_t\dot{P}^e_{t+1}$$

$$(3) \quad {}_t r^e_{t+1} = \bar{r},$$

where $\dot{P}_{t+1}$ is the actual rate of inflation in period t+1, ${}_t\dot{P}^e_{t+1}$ is the expected rate of inflation for t+1, conditional on information available in t, μ_{t+1} is a random disturbance term, i_t is the nominal rate of interest, ${}_t r^e_{t+1}$ is the real rate expected to prevail in period t+1 and $\bar{r}$ is a constant. Equation (1) decomposes inflation into its expected (${}_t\dot{P}^e_{t+1}$) and unexpected (μ_{t+1}) components. Assuming rational expectations, as Fama does, the latter component should behave as white noise. Equation (2) requires that the nominal yield on a financial asset be identically equal to the expected real rate on capital plus inflationary expectations. Equation (3) represents the hypothesis that the expected real rate is a constant.

In Fama's treatment there is only one endogenous variable ($\dot{P}_{t+1}$), while there are three exogenous variables (i_t, ${}_t r^e_{t+1}$ and ${}_t\dot{P}^e_{t+1}$). The model is sufficiently identified as to be easily estimated with ordinary least squares. The unique twist provided by Fama is to use equations (2) and (3) to eliminate the unobservable inflationary expectations variable (${}_t\dot{P}^e_{t+1}$). Solving these equations for ${}_t\dot{P}^e_{t+1}$ and substituting into equation (1) yields

$$(4) \quad \dot{P}_{t+1} = -\bar{r} + i_t + \mu_{t+1}.$$

The empirical counterpart of equation (4), which Fama estimated with ordinary least squares, is

$$(5) \quad \dot{P}_{t+1} = \beta_0 + \beta_1 i_t + \nu_t.$$

If all the restrictions of the above model hold, then estimates of β_1 should not be statistically different from unity and the residuals (ν_t) should be independent and identically distributed with a zero mean and finite variance. Fama found that he could not reject the model. The estimate of β_1 was not different from unity and the residuals, as well as _ex post_ real rates, were not serially correlated.[3/]

Relaxing the Assumption of a Constant Expected Real Rate

Fama's inability to reject the hypothesis that the expected real rate is constant attracted much criticism. Carlson (1977) and Nelson and Schwert (1977), for example, presented evidence allowing them to reject this hypothesis. Our concern is not with the actual empirical evidence, but, more importantly, with what becomes of Fama's model when this restriction is relaxed.

As an alternative to the assumption of a constant real rate, Lahari (1976) and Dwyer (1981) each postulate that the expected real rate fluctuates randomly around a constant:

$$(3') \quad {}_t r^e_{t+1} = \bar{r} + \eta_t .$$

Substituting this relationship into equation (2) yields

$$(2') \quad i_t = \bar{r} + {}_t\dot{P}^e_{t+1} + \eta_t .$$

If equation (2') is solved for ${}_t\dot{P}^e_{t+1}$ and substituted into equation (1), the model yields the estimable equation,

$$(4') \quad \dot{P}_{t+1} = -\bar{r} + i_t - \eta_t + \mu_{t+1} .$$

Estimating equation (4') with ordinary least squares, however, yields biased coefficient estimates because of the correlation between the independent variable, i_t, and the η_t part of the composite disturbance term implied by equation (2').[4/] Thus, if the expected real rate does fluctuate randomly about a constant, Fama's coefficient estimate on the variable i_t is biased downwards.

Fama himself recently recanted on the position that the expected real rate is constant.[5/] The hypothesis advocated in Fama (1981) is that the expected real rate is a random walk, or,

$$(3'') \quad {}_{t}r^{e}_{t+1} = {}_{t-1}r^{e}_{t} + \phi_{t+1}.$$

When this relationship is substituted into equation (2), the interest rate is again correlated with the random element of the expected real rate (ϕ_{t+1}):

$$(2'') \quad i_t = {}_{t-1}r^{e}_{t} + {}_{t}\dot{P}^{e}_{t+1} + \phi_{t+1}.$$

Thus, the true model linking next period's inflation to this period's interest rate, under the random walk assumption, is

$$(4'') \quad \dot{P}_{t+1} = - {}_{t-1}r^{e}_{t} + i_t - \phi_{t+1} + \mu_{t+1}.$$

Although Fama does not estimate equation (4'') with ordinary least squares, the problem remains that the econometric technique used is premised on the assumption that the random element of the expected real rate (ϕ_{t+1}) is independent of the exogenous variable (i_t).[6/] Equation (2'') reveals that this assumption is violated in such a model, raising doubts about Fama's most recent empirical findings.

The Nonstochastic Link between Nominal Yields

Relaxing Fama's restrictive hypothesis with regard to the expected real rate raises serious econometric questions about his estimation technique and results. It is also important to recognize that relaxing Fama's restriction that the nominal returns to bills and capital be equal yields similar conclusions. Equation (2) in Fama's framework, with the exclusion of any disturbance term, requires these two ex ante nominal yields always to be equal. Thus, the two investment alternatives are, by assumption, perfect substitutes.

The periodicity of Fama's analysis is crucially important in this regard. Fama has emphasized monthly data in his empirical work.[7/] Consequently, equation (2) requires that short-term bills and capital be perfect substitutes on a monthly basis: Ex ante real and nominal returns on the two assets are, by assumption, identical. Alternatively, ex post returns on these assets will be identical and these ex post returns on each will differ from ex ante returns only by the magnitude of unanticipated inflation (μ_{t+1}).

Such a restriction is very limiting. It ignores risk differentials and shifts in relative risk. Further, it assumes that transaction costs are irrelevant: If inflation is expected to rise rapidly next month, individuals are assumed to immediately sell their financial assets driving up nominal interest rates until they equal the expected monthly nominal yield for capital. This behavior is necessary even if, for example, inflation is expected to be much lower the following month. In Fama's model nothing really impinges on the investment decision except next period's expectations: ${}_t r^e_{t+1}$ and ${}_t \dot{P}^e_{t+1}$.

This restriction is empirically relaxed by including a disturbance term (ε_t) in the relationship linking the *ex ante* nominal rates. This yields a model of the form

$$(1) \quad \dot{P}_{t+1} = {}_t\dot{P}^e_{t+1} + \mu_{t+1}$$

$$(2''') \quad i_t = {}_t r^e_{t+1} + {}_t\dot{P}^e_{t+1} + \varepsilon_t$$

$$(3) \quad {}_t r^e_{t+1} = \bar{r} \,.$$

In the framework implied by equations (1), (2''') and (3), the *ex ante* nominal bill rate can momentarily diverge from the *ex ante* nominal rate on capital. Requiring that $E(\varepsilon_t)=0$, however, imposes a "long-run" condition of perfect substitutability. This is more in line with Fisher's original analysis and, it seems, would be acceptable to most macroeconomists.

This modest alteration again calls into question Fama's empirical findings. Substituting equations (2''') and (3) into (1) yields a relationship that will not provide unbiased coefficient estimates when estimated with ordinary least squares. Again this is due to the correlation between i_t and ϵ_t. Moreover, correctly estimating the model, for example with a simultaneous equations or instrumental variables approach, requires observations of the exogenous variable ${}_t\dot{P}^e_{t+1}$. Thus, the unique twist of Fama's original model, dispensing with the need of observations on expected inflation, is nullified.

Conclusion

Fama's original model includes three equations with one endogenous variable and three exogenous variables. Proper substitution allowed him to avoid the thorny problem of constructing a viable measure of expected inflation. This insight represents an innovative approach in the long history of estimating the relationship between nominal interest rates on financial assets and expected inflation.

The purpose here has been to point out the limitations of this approach--limitations which Fama recently has made binding and important. In general, If either the *ex ante* real or market interest rate is made endogenous, ordinary least squares estimates (such as Fama's) are biased. This is true, for example, when real interest rates are assumed to be nonconstant, or if nominal yields on capital and bonds are not constrained to month-to-month equality. In addition, correcting for these estimation problems through generally practiced simultaneous equation techniques, requires observations on expected inflation in at least one stage of the estimation process--the very problem Fama sought to circumvent. The upshot is that one can not avoid the problem of generating such measures when studying inflation and interest rates in a realistic setting.

Footnotes

1/ Eugene F. Fama, "Short-Term Interest Rates as Predictors of Inflation," American Economic Review, June 1975, 65, p. 282.

2/ Early examples are John A. Carlson, "Short-Term Interest Rates as Predictors of Inflation: Comment," American Economic Review, June 1977, 67, pp. 469-75; Douglas Joines, "Short-Term Interest Rates as Predictors of Inflation: Comment," American Economic Review, June 1977, 67, pp. 476-79; and Charles R. Nelson and G. William Schwert, "Short-Term Interest Rates as Predictors of Inflation: On Testing the Hypothesis that the Real Rate of Interest is Constant," American Economic Review, June 1977, 67, pp. 478-486. See also Fama's rejoinder to his critics, "Interest Rates and Inflation: The Message in the Entrails," American Economic Review, June 1977, 67, pp. 487-496.

3/ Fama really is testing whether his inflationary expectations proxy is an unbiased predictor of next period's inflation and not the joint hypothesis that the expected real rate is constant and that interest rates move one-for-one with inflationary expectations. These latter are essentially maintained hypotheses that validate his estimation procedure.

4/ Theil (1971), pp. 429-83 provides an informal analysis of this problem.

5/ Eugene F. Fama, "Stock Returns, Real Activity, Inflation, and Money," American Economic Review September 1981, 71, pp. 545-565.

6/ The econometric technique used by Fama is described in Craig F. Ansley, "Signal Extraction in Finite Series and the Estimation of Stochastic Regression Coefficients," Proceedings of the American Statistical Association, Business and Economics Section, 1980. See also Eugene F. Fama and Michael R. Gibbons, "Inflation, Real Returns, and Capital Investment," working paper no. 39, Center for Research in Security Prices, Graduate School of Business, University of Chicago, December 1980.

7/ If bond and capital yields are not equal on a monthly basis, then Fama has a time aggregation estimation problem since the model's restrictions are essentially false. Improper temporal aggregation can lead to loss of efficiency and, under special conditions, biased estimated coefficients. See, for example, C. Moriguchi, "Aggregation Over Time in Macroeconomic Relations," International Economic Review, October 1970, pp. 427-440.

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